

VIOLATIONS OF A NEW INEQUALITY FOR CLASSICAL FIELDS

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ABSTRACT

Two entangled photons incident upon two distant interferometers can give a coincidence counting rate that depends nonlocally on the sum of the phases of the two interferometers. It has recently been shown that experiments of this kind may violate a simple inequality that must be satisfied by any classical or semi-classical field theory. The inequality provides a graphic illustration of the lack of objective realism of the electric field. The results of a recent experiment which violates this inequality and in which the optical path length between the two interferometers was greater than 100 m are briefly described.

INTRODUCTION

It has been shown^{1,2} that two-photon interferometer experiments can violate Bell's inequality³ and a number of experiments⁴⁻⁷ have demonstrated effects of that kind. Several experiments^{4,6} based upon the two-photon interferometer of Ref. 1 have not, however, violated Bell's inequality due to the limited visibility (50%) of the interference fringes that results when the resolving time of the photon detectors and electronics is not sufficiently fast.

Those experiments may⁸, however, violate a surprisingly simple inequality that must be satisfied by any classical or semi-classical field theory. The inequality follows directly from the assumption that the classical field has some well-defined value and thus illustrates the lack of objective realism exhibited by the quantum-mechanical field.

This paper will briefly review the nature of two-photon interferometry and then derive the new inequality; the derivation closely follows that of Ref. 8. Some additional details of the derivation that are not contained in Ref. 8 but are required for applications to actual experiments are presented in the Appendix. The results of a recent two-photon interferometer experiment performed over a distance of 100 meters will be briefly described. Finally, some comments will be made with regard to the connection between uncertainty relations and inequalities of this type.

TWO-PHOTON INTERFEROMETRY

The experiments of interest^{4,6,7} are outlined in Figure 1. Two coincident photons are emitted by parametric down-conversion and travel in different directions toward two identical interferometers. Each interferometer contains a shorter and a longer path, and the difference ΔT in transit times over the two paths is taken to be much larger than the coherence time of the photons. Nevertheless, interference between the quantum-mechanical amplitudes for the photons to have both traveled the shorter paths or the longer paths produces a modulation in the coincidence counting rate R_c given¹ by

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$$R_c = \frac{1}{4} R_{c0} \cos^2 \left(\frac{\theta_1 + \theta_2 + \omega_0 \Delta T}{2} \right) \quad (1)$$

Here R_{c0} is the coincidence rate with the beam splitters removed, θ_1 and θ_2 are phase-shifts introduced into the two longer paths, and ω_0 is the frequency of the pump laser. Eq. (1) violates Bell's inequality but is only valid if the resolution of the coincidence measurements is better than ΔT . The maximum visibility is 50% for time resolutions much worse than ΔT .

There has been some question as to whether or not the experiments with visibilities of 50% or less are nevertheless inconsistent with any semi-classical field theory. Ou and Mandel⁹ have suggested that that is the case but counter-examples to their argument have been given by Carmichael¹⁰ and by Chiao and Kwiat¹¹. Although their semi-classical models are able to reproduce the modulation in the coincidence rate, they are not able to represent the fact that the photons are known from other experiments¹² to be coincident to within a time interval much smaller than ΔT . That provides the physical basis for the inequalities derived below.

BASIC INEQUALITY

The basic inequality that must be satisfied by any classical field is based on Cauchy's inequality¹³, which follows from the fact that

$$(a - b)^2 \geq 0 \quad (2)$$

where a and b are any two real numbers. Multiplying the two factors and rearranging gives Cauchy's inequality:

$$2ab \leq a^2 + b^2 \quad (3)$$

When a and b are complex it is still the case that

$$|ab| = |a| |b| \leq \frac{|a|^2 + |b|^2}{2} \quad (4)$$

The modulation of the coincidence rate will be found to be proportional to the quantity Q defined by

$$Q \equiv \langle |E_1^*(t) E_2^*(t) E_2(t - \Delta T) E_1(t - \Delta T)| \rangle \quad (5)$$

Here E_1 and E_2 refer to the fields at the positions of detectors 1 and 2 (which will be assumed to be equidistant from the source) with the beam splitters removed and $\langle \rangle$ denotes an average over a long time interval.

It should be emphasized from the start that the angular brackets denote an average over time and not an ensemble average. That is what the experiments actually measure, since the results from a single system are simply averaged over time. In addition, no assumption of ergodicity is required in the proof that follows; the average over an ensemble is not considered and it therefore makes no difference whether or not the time average is equivalent to an ensemble average. It will also be found that the proof does not assume stationarity, either.

The basic inequality can be obtained by choosing

$$a = E_1^*(t) E_2(t - \Delta T) \quad (6)$$

$$b = E_2^*(t) E_1(t - \Delta T) \quad (7)$$

Inserting eqs. (6) and (7) into eq. (4) gives

$$\begin{aligned} & \langle |E_1^*(t) E_2^*(t) E_2(t - \Delta T) E_1(t - \Delta T)| \rangle \\ & \leq \langle E_1^*(t) E_2^*(t - \Delta T) E_2(t - \Delta T) E_1(t) \rangle / 2 \\ & + \langle E_2^*(t) E_1^*(t - \Delta T) E_1(t - \Delta T) E_2(t) \rangle / 2 \end{aligned} \quad (8)$$

The physical significance of the above inequality can be seen in Figure 2, in which both fields $E_1(t)$ and $E_2(t)$ correspond to narrow pulses emitted at the same time. If E_1 is evaluated at time t and E_2 is evaluated at time $t + \Delta T$, as illustrated by the arrows in the figure, then one or the other of the fields must be zero and their product vanishes. The right-hand-side of eq. (8) is then zero, which requires that the left-hand-side also vanish. Although this inequality may seem trivial in nature, it is a consequence of the fact that the classical fields are well-defined (complex) numbers and the inequality is violated by quantum fields, as will be discussed below.

INEQUALITY FOR THE VISIBILITY

The inequality of eq. (8) can be used to set a limit on the amount of modulation that can occur in a classical treatment of the two-photon interferometer experiments. Once again, let $E_1(t)$ be the classical field that would arrive at detector 1 in the absence of the two beam splitters and assume for the moment that the half-width w of the coincidence window is negligibly small. The corresponding coincidence rate as a function of the time offset τ is then

$$\begin{aligned} R_{co}(\tau) &= \eta \langle I_1(t) I_2(t + \tau) \rangle \\ &= \eta \langle E_1^*(t) E_2^*(t + \tau) E_2(t + \tau) E_1(t) \rangle \end{aligned} \quad (9)$$

where I_1 and I_2 are the intensities of the two beams and the constant η is related to the detection efficiencies and w . With the insertion of the two beam splitters, the total electric field $E_{T1}(t)$ at detector 1 becomes

$$E_{T1} = \frac{1}{2} [E_1(t) + e^{i\theta_1} E_1(t - \Delta T)] \quad (10)$$

A similar expression exists for the total field at detector 2 and the classical coincidence rate R_c with the beam splitters inserted and $\tau = 0$ is given by

$$R_c = \frac{1}{16} \eta \langle | [E_1(t) + e^{i\theta_1} E_1(t - \Delta T)] [E_2(t) + e^{i\theta_2} E_2(t - \Delta T)] |^2 \rangle \quad (11)$$

Multiplying out all the factors in eq.(11) gives a total of sixteen terms:

$$\begin{aligned}
R_c = & \frac{1}{16} \eta \langle E_1^*(t) E_2^*(t) E_1(t) E_2(t) \\
& + e^{i\theta_2} E_1^*(t) E_2^*(t) E_1(t) E_2(t - \Delta T) \\
& + e^{i\theta_1} E_1^*(t) E_2^*(t) E_1(t - \Delta T) E_2(t) \\
& + e^{i[\theta_1 + \theta_2]} E_1^*(t) E_2^*(t) E_1(t - \Delta T) E_2(t - \Delta T) \\
& + e^{-i\theta_2} E_1^*(t) E_2^*(t - \Delta T) E_1(t) E_2(t) \\
& + E_1^*(t) E_2^*(t - \Delta T) E_1(t) E_2(t - \Delta T) \\
& + e^{i[\theta_1 - \theta_2]} E_1^*(t) E_2^*(t - \Delta T) E_1(t - \Delta T) E_2(t) \\
& + e^{i\theta_1} E_1^*(t) E_2^*(t - \Delta T) E_1(t - \Delta T) E_2(t - \Delta T) \\
& + e^{-i\theta_1} E_1^*(t - \Delta T) E_2^*(t) E_1(t) E_2(t) \\
& + e^{i[\theta_2 - \theta_1]} E_1^*(t - \Delta T) E_2^*(t) E_1(t) E_2(t - \Delta T) \\
& + E_1^*(t - \Delta T) E_2^*(t) E_1(t - \Delta T) E_2(t) \\
& + e^{i\theta_2} E_1^*(t - \Delta T) E_2^*(t) E_1(t - \Delta T) E_2(t - \Delta T) \\
& + e^{i[-\theta_1 - \theta_2]} E_1^*(t - \Delta T) E_2^*(t - \Delta T) E_1(t) E_2(t) \\
& + e^{-i\theta_1} E_1^*(t - \Delta T) E_2^*(t - \Delta T) E_1(t) E_2(t - \Delta T) \\
& + e^{-i\theta_2} E_1^*(t - \Delta T) E_2^*(t - \Delta T) E_1(t - \Delta T) E_2(t) \\
& + E_1^*(t - \Delta T) E_2^*(t - \Delta T) E_1(t - \Delta T) E_2(t - \Delta T) \rangle
\end{aligned} \tag{12}$$

As suggested by eq. (1), the experiments can be performed in such a way as to measure the averaged coincidence rate as a function of $\theta_r = \theta_1 + \theta_2$:

$$\bar{R}_c(\theta_r) = \frac{1}{2\pi} \int_0^{2\pi} d\theta_1 \int_0^{2\pi} d\theta_2 R_c(\theta_1, \theta_2) \delta(\theta_1 + \theta_2 - \theta_r) \tag{13}$$

The averages over θ_1 and θ_2 were explicitly performed in one of the experiments⁶. In the remaining experiments the individual phases were not directly measured and had essentially random values from one run to the next, since variations in the temperature of the laboratory would have shifted the phase of both interferometers by several fringes from one day to the next. Thermal drifts during the course of an experimental run would have a similar effect on the individual phases while leaving the modulation of the coincidence rate unaltered.

In any event, the terms in eq. (12) with phase factors of $\exp(i\theta_1)$, $\exp(i\theta_2)$, $\exp[i(\theta_1 - \theta_2)]$, etc., average to zero, leaving only those terms with no phase dependence or a dependence on $\theta_1 + \theta_2$. The remaining terms can be written as

$$\begin{aligned}
\bar{R}_c = & \frac{1}{8} \eta \langle E_1^*(t) E_2^*(t) E_2(t) E_1(t) \rangle \\
& + \frac{1}{8} \eta \langle E_1^*(t) E_2^*(t - \Delta T) E_2(t - \Delta T) E_1(t) \rangle \\
& + \frac{1}{16} \eta [e^{i\theta_r} \langle E_1^*(t) E_2^*(t) E_2(t - \Delta T) E_1(t - \Delta T) \rangle + c.c.]
\end{aligned} \tag{14}$$

where the average over a long time interval ensures that

$$\begin{aligned}
& \langle E_1^*(t - \Delta T) E_2^*(t - \Delta T) E_2(t - \Delta T) E_1(t - \Delta T) \rangle \\
& = \langle E_1^*(t) E_2^*(t) E_2(t) E_1(t) \rangle
\end{aligned} \tag{15}$$

and the symmetry of the two beams gives

$$\begin{aligned} & \langle E_1^*(t) E_2^*(t - \Delta T) E_2(t - \Delta T) E_1(t) \rangle \\ & = \langle E_2^*(t) E_1^*(t - \Delta T) E_1(t - \Delta T) E_2(t) \rangle \end{aligned} \quad (16)$$

The assumption inherent in eq. (16) is not essential and can be avoided by simply replacing $R_{c0}(\Delta T)$ with $[R_{c0}(\Delta T) + R_{c0}(-\Delta T)]/2$ in what follows.

The maximum and minimum coincidence rates from eq. (14) satisfy

$$\begin{aligned} R_{\max} & \leq \frac{1}{8} \eta \langle E_1^*(t) E_2^*(t) E_2(t) E_1(t) \rangle \\ & + \frac{1}{8} \eta \langle E_1^*(t) E_2^*(t - \Delta T) E_2(t - \Delta T) E_1(t) \rangle \\ & + \frac{1}{8} \eta \langle |E_1^*(t) E_2^*(t) E_2(t - \Delta T) E_1(t - \Delta T)| \rangle \end{aligned} \quad (17)$$

$$\begin{aligned} R_{\min} & \geq \frac{1}{8} \eta \langle E_1^*(t) E_2^*(t) E_2(t) E_1(t) \rangle \\ & + \frac{1}{8} \eta \langle E_1^*(t) E_2^*(t - \Delta T) E_2(t - \Delta T) E_1(t) \rangle \\ & - \frac{1}{8} \eta \langle |E_1^*(t) E_2^*(t) E_2(t - \Delta T) E_1(t - \Delta T)| \rangle \end{aligned} \quad (18)$$

The visibility is defined as usual by

$$V = \frac{R_{\max} - R_{\min}}{R_{\max} + R_{\min}} \quad (19)$$

Using the inequality of eq. (8) and expressing the right-hand-side in terms of $R_{c0}(\Delta T)$ gives

$$V \leq \frac{R_{c0}(\Delta T)}{R_{c0}(0) + R_{c0}(\Delta T)} \quad (20)$$

Eq. (20) gives the maximum visibility that can occur in any classical field theory and gives zero modulation for the case in which the fields correspond to coincident pulses.

If the experiments are performed using detectors with limited time responses and large coincidence windows, as is often the case, then the above inequality can be generalized to

$$V \leq \frac{\int_{\Delta T/2}^{\infty} R_{c0}(\tau) d\tau + \frac{1}{2} \int_{\Delta T/2}^{3\Delta T/2} R_{c0}(\tau) d\tau}{2 \int_0^{\infty} R_{c0}(\tau) d\tau} \quad (21)$$

as is shown in the Appendix. R_{c0} is again the coincidence rate that would be obtained using detectors with a negligible time response and a negligibly-small window.

COMPARISON WITH EXPERIMENT

Earlier experiments¹² have shown that the down-converted photons are coincident to within a time interval much less than the value of ΔT in at least three^{4,6,7} of the two-photon interferometer experiments, in which case the inequalities of eqs. (20) or (21) show that there is no classical or semi-

classical field theory consistent with all of the available observations.

The author has recently completed an experiment ⁶ in which the optical path length between the two interferometers was larger than 100 meters. The main goal of the experiment was to investigate these effects in the limit of large distances. Furry has suggested that the collapse of the wavefunction may be degraded in some way when it occurs over sufficiently large distances, leading to an eventual modification of the quantum-theory predictions. The visibility of the interference pattern observed agreed with that expected from the quantum theory to within the experimental uncertainty of 4% and violated the inequality of eq. (21) by four standard deviations. This provides some indication that the collapse of the wavefunction is unaffected even when it occurs over relatively large optical path lengths.

CLASSICAL MODELS

In the classical models suggested by Carmichael⁹ and by Chiao and Kwiat¹⁰, the fields E_1 and E_2 have well-defined frequencies that sum to the pump laser frequency for a time interval larger than ΔT or the time resolution of the coincidence circuits. In that case the coincidence rate of eq. (14) simplifies to

$$\overline{R_c} = \frac{1}{4} R_{co} \left[\cos^2 \left(\frac{\theta_T + \omega_0 \Delta T}{2} \right) + 1/2 \right] \quad (22)$$

This differs from the quantum-mechanical result by the additional factor of 1/2 and corresponds to a visibility of 50%.

Such models cannot simultaneously localize the fields into coincident pulses whose widths are less than ΔT , however. Any classical model that does would have the visibility reduced accordingly as required by the inequalities of eqs. (20) or (21).

VIOLATION OF THE INEQUALITY IN QUANTUM OPTICS

The intensity operator is given by $I(t) = E^-(t)E^+(t)$, where E^+ and E^- are the positive and negative-frequency components of the electric field operator¹³. As a result, the quantum-mechanical equivalent of eq. (8) is

$$\begin{aligned} & | \langle E_1^-(t) E_2^-(t) E_2^+(t - \Delta T) E_1^+(t - \Delta T) \rangle | \\ & \leq \langle E_1^-(t) E_2^-(t - \Delta T) E_2^+(t - \Delta T) E_1^+(t) \rangle / 2 \\ & + \langle E_2^-(t) E_1^-(t - \Delta T) E_1^+(t - \Delta T) E_2^+(t) \rangle / 2 \end{aligned} \quad (23)$$

It has already been noted¹ that in experiments of this kind the coincidence of the photons requires

$$E_1^+(t) E_2^+(t \pm \Delta T) = 0 \quad (24)$$

while conservation of energy in the parametric down-conversion process requires that

$$E_1^+(t - \Delta T) E_2^+(t - \Delta T) = e^{i(\omega_1 + \omega_2)\Delta T} E_1^+(t) E_2^+(t) \quad (25)$$

where the sum of the two photon frequencies ω_1 and ω_2 is equal to ω_0 . (Eq. (24) is only valid when ΔT is small compared to the pump laser coherence time.) Inserting eq. (24) into the right-hand-side of eq. (23) gives zero, whereas inserting eq. (25) into the left-hand-side gives

$\langle E_1^-(t) E_2^-(t) E_2^+(t) E_1^+(t) \rangle$, which is the product of the individual beam

intensities and a nonzero quantity. Thus the inequality is violated in quantum optics.

The quantum-mechanical situation is shown in Figure 3. The field corresponds to an entangled state in which there is a superposition of times at which the pair of photons may have been emitted, as indicated by the existence of both the solid and dotted curves. Although the product of E_1^* and E_2^* at two different times is zero, that does not imply that the left-hand-side of eq. (22) must vanish. Equations (24) and (25) would be logically inconsistent if the fields were well-defined complex numbers and the violation of this inequality provides a graphic demonstration of the lack of objective realism of the electric field.

CONNECTION WITH UNCERTAINTY RELATIONS

The main topics of this conference are squeezing and uncertainty relations. It may thus be useful to make some general comments about the connection between the inequality derived above and the uncertainty relations associated with the quantized field.

The inequality derived above is a result of the fact that the fields are not just complex numbers and thus have no well-defined value. In particular, the field operators are non-commuting and satisfy

$$[A_\mu(x), A_\nu(x')] = -ich\delta_{\mu\nu}D(x - x') \quad (26)$$

A variety of uncertainty relations can be derived from this commutation relation, which illustrates the fact that the quantized field has no well-defined value. As a result, there is an unavoidable uncertainty in the left-hand-side of the classical inequality and this uncertainty is evidently large enough that the left-hand-side can exceed the right-hand-side. Thus it seems apparent that the violations of these classical inequalities in quantum optics are related to the uncertainty relations for the quantized fields. More detailed uncertainty relations for the actual quantities involved in the classical inequality could be derived, if desired.

SUMMARY

Two-photon interferometer experiments with a sufficiently large visibility will violate Bell's inequality and are thus inconsistent with any local hidden-variable theory. Those experiments with smaller visibilities may nevertheless violate an inequality for classical fields if the degree of coincidence of the photon counts is taken into account. A recent two-photon interferometer experiment with a large optical path length between the two interferometers gave a visibility in good agreement with the quantum theory and also violated the classical inequality, indicating that the effects observed were quantum-mechanical in nature.

APPENDIX

When finite coincidence windows are used, Eq. (9) must be replaced by

$$R'_{co} = \eta' \int_{-w}^w \langle I_1(t) I_2(t + \tau) \rangle d\tau \quad (27)$$

The modulation in the coincidence rate then involves

$$Q' = \frac{1}{8} \eta' \int_{-w}^w d\tau \langle |E_1^*(t) E_2^*(t+\tau) E_2(t - \Delta T + \tau) E_1(t - \Delta T)| \rangle \quad (28)$$

The range of the integral can be divided into two regions depending on the value of $|\tau|$. For $|\tau| \leq \Delta T/2$, a and b can be chosen as

$$\begin{aligned} a &= E_1^*(t) E_2(t - \Delta T + \tau) \\ b &= E_2^*(t + \tau) E_1(t - \Delta T) \end{aligned} \quad (29)$$

and the analysis proceeds as in the text. For $|\tau| \geq \Delta T/2$, a and b are chosen instead as

$$\begin{aligned} a &= E_1^*(t) E_2^*(t + \tau) \\ b &= E_2(t - \Delta T + \tau) E_1(t - \Delta T) \end{aligned} \quad (30)$$

The inequality of eq. (21) then results from the use of eq. (8) in the limit that w is much larger than ΔT .

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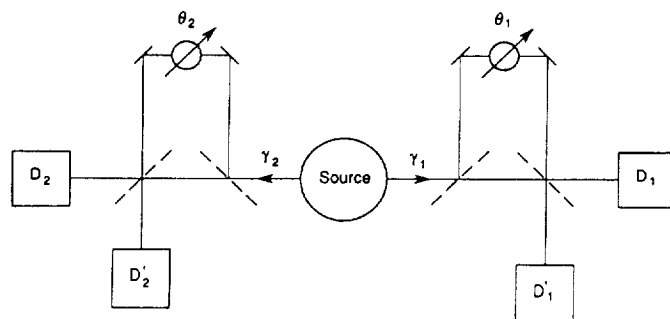


Figure 1 Two-photon interferometer.

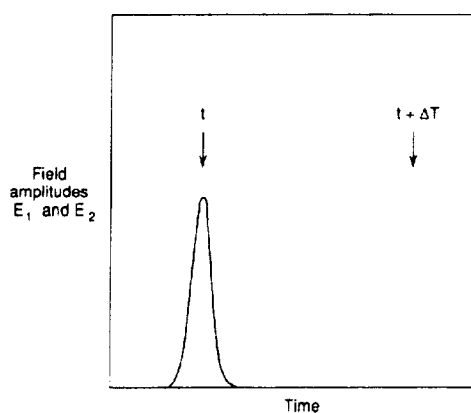


Figure 2 A pair of classical coincident pulses.

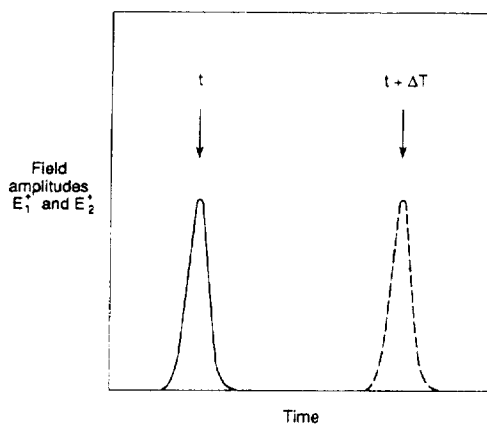


Figure 3 Quantum-mechanical field corresponding to an entangled pair of coincident photons, with a superposition of times at which the pair may have been emitted.